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Local convergence in measure on semifinite von Neumann algebras

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Abstract

Suppose that M is a von Neumann algebra of operators on a Hilbert space H and τ is a faithful normal semifinite trace on M. The set M̃ of all τ-measurable operators with the topology tτ of convergence in measure is a topological *-algebra. The topologies of τ-local and weakly τ-local convergence in measure are obtained by localizing t τ and are denoted by tτ1 and twτ1, respectively. The set M̃ with any of these topologies is a topological vector space. The continuity of certain operations and the closedness of certain classes of operators in M̃ with respect to the topologies t τ1 and twτ1 are proved. S.M. Nikol'skii's theorem (1943) is extended from the algebra B(H) to semifinite von Neumann algebras. The following theorem is proved: For a von Neumann algebra M with a faithful normal semifinite trace τ, the following conditions are equivalent: (i) the algebra M is finite; (ii) twτ1 = tτ1; (iii) the multiplication is jointly tτ1-continuous from M̃ × M̃ to M̃; (iv) the multiplication is jointly twτ1- continuous from M̃ × M̃ to M̃; (v) the involution is t τ1-continuous from M̃ to M̃.

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